$$x_1=\left(\frac{R_1}{2}+\sqrt{m^5+\frac{R_1^2}{4}}\right)^{\frac{1}{5}}+\left(\frac{R_1}{2}-\sqrt{m^5+\frac{R_1^2}{4}}\right)^{\frac{1}{5}}=s_1+s_2,$$

$x_2=es_1+e^4s_4$; $x_3=e^2s_1+e^3s_4$; $x_4=e^3s_1+e^2s_4$, and $x_5=e^4s_1+es_4$, when e represents the imaginary fifth root of unity.

The other ten values of x can be found by substituting R_2 and R_3 for R_1 above.

II. **Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.**

Let $x=y+z$.

$\therefore\ y^{15}+z^{15}+(yz+m)[15(y^{13}+z^{13})+(105yz+90m)(y''+z'')+(455y^2z^2+715myz+275m^2)(y^9+z^9)+(1365y^3z^3+2925my^2z^2+2025m^2yz+450m^3)(y^7+z^7)+(3003y^4z^4+7722my^3z^3+7128m^2y^2z^2+2772m^3yz+378m^4)(y^5+z^5)+(5005y^5z^5+14300my^4z^4+15400m^2y^3z^3+7700m^3y^2z^2+1750m^4yz+140m^5)(y^3+z^3)+(6435y^6z^6+19305my^5z^5+22275m^2y^4z^4+12375m^3y^3z^3+3375m^4y^2z^2+405m^5yz+15m^6)(y+z)]+2r=0.$

$\therefore y^{15}+z^{15}=-2r$, $yz=-m$. Let $y^{15}=a$, $z^{15}=b$.

$\therefore a+b=-2r$, $ab=-m^{15}$. $\therefore a$ and b are the roots of $t^2+2rt-m^{15}=0$.

Let $\beta=$an imaginary fifteenth root of unity and also let $a^{\frac{1}{15}}=c$, $b^{\frac{1}{15}}=d$.

$\therefore$ The roots are $c+d$, $\beta c+\beta^{14}d$, $\beta^{14}c+\beta d$, $\beta^2c+\beta^{13}d$, $\beta^{13}c+\beta^2d$, $\beta^3c+\beta^{12}d$, $\beta^{12}c+\beta^3d$, $\beta^4c+\beta^{11}d$, $\beta^{11}c+\beta^4d$, $\beta^5c+\beta^{10}d$, $\beta^{10}c+\beta^5d$, $\beta^6c+\beta^9d$, $\beta^9c+\beta^6d$, $\beta^7c+\beta^8d$, $\beta^8c+\beta^7d$.

Also solved by J. Scheffer.

206. **Proposed by L. E. NEWCOMB, Los Gatos, Cal.**

The product of a certain pair of roots of $x^4+ax^3+bx^2+amx+m^2=0$, is equal to the product of the remaining pair.

I. **Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.**

Let x_1, x_2, x_3, x_4 be the roots. Then $x_1+x_2+x_3+x_4=-a$; $x_1x_2x_3+x_1x_2x_4+x_1x_3x_4+x_2x_3x_4=-am$; $x_1x_2x_3x_4=m^2$.

$\therefore m(x_1+x_2+x_3+x_4)=x_1x_2(x_3+x_4)+x_3x_4(x_1+x_2)=-am.$

$\therefore x_1x_2=x_3x_4=m.$

The same is true if $x_1x_3=x_2x_4=m$, $x_1x_4=x_2x_3=m$.

II. **Solution by J. SCHEFFER, Kee Mar College, Hagerstown, Md.**

From the theory of equations, we have $\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta=-am$, or $\alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta)=-am$, but $\gamma\delta=\alpha\beta$.

$\therefore \alpha\beta(\alpha+\beta+\gamma+\delta)=-am$. Since $\alpha+\beta+\gamma+\delta=-a$, $-\alpha\beta a=-am$.

$\therefore \alpha\beta=m=\gamma\delta$.

III. Solution by G. W. GREENWOOD, M. A. (Oxon), Lebanon, Ill.

$\Pi(\alpha\beta-\gamma\delta)=\alpha\beta\gamma\delta(\Sigma\alpha)^2-(\Sigma\alpha\beta\gamma)^2=m^2(-a)^2-(-am)^2=0.$

Hence the product of any one pair of the roots equals that of the remaining pair.

Solved by A. H. Holmes by actual computation of the roots.
Also solved by the Proposer.

207. Proposed by A. J. PAULSEN, San Francisco, Cal.

Solve $x^4+y^4=2x^2y^2$; $x+y=a$.

Solution by EDWIN L. RICH, Lehigh University, and A. H. HOLMES, Brunswick, Maine.

Writing the first of these equations in the form $(x+y)^2(x-y)^2=0$, it is easily seen, since $x+y=a$, that the four values of x are ∞, $-\infty$, $\frac{1}{2}a$, $\frac{1}{2}a$; and those of y are $-\infty$, ∞, $\frac{1}{2}a$, $\frac{1}{2}a$.

Also solved by G. B. M. Zerr, G. W. Greenwood, M. E. Graber, and J. Scheffer.

GEOMETRY.

233. Proposed by S. F. NORRIS, Professor of Mathematics, Baltimore City College, Baltimore, Md.

If from any point on a circle circumscribed about a triangle perpendiculars are dropped to the sides of the triangle, the feet of these perpendiculars lie on a line. [Ashton's *Plane and Solid Analytic Geometry*, page 87, 11th example].

Remark by G. W. GREENWOOD, M. A. (Oxon), Lebanon, Ill.

See solution of Geometry Problem number 184, August-September, 1902.

234. Proposed by M. E. GRABER, A. B., Instructor in Mathematics and Physics in Heidelberg University, Tiffin, Ohio.

Find the curve which is reciprocal to a circle and define it as a locus.

I. Solution by the PROPOSER.

If A be the center of the given circle, P any doint on it; x and a lines corresponding to P and A; and X the point ax, $AP=\dfrac{OA.OX}{Ox}\sin ax$. Denoting OX by z the ratio $\sin ax:\sin zx$ is constant. The reciprocal to a circle is then the envelope of a line x which divides the angle between a fixed line a and a variable line z passing through a fixed point O, into parts whose sines are in a constant ratio. Defined as a locus, the reciprocal curve to a circle is the path of a point which moves so that its distance from a fixed point varies as its distance from a fixed straight line. [Lachlan's *Modern Pure Geometry*.]

II. Solution by G. W. GREENWOOD, M. A. (Oxon), Professor of Mathematics and Astronomy in McKendree College, Lebanon, Ill.

See Russell's *Pure Geometry*, Chapter VIII, §11, or Salmon's *Conic Sections*, §308. The following is an analytic solution. Call the center of the given circle C and its radius a. Call the center of the circle of reciprocation O. De-